Zero mode contribution in quarkonium correlators and in-medium properties of heavy quarks

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Abstract. We calculate the low energy contribution to quarkonium correlators in Euclidean time in lattice QCD. This contribution was found to give the dominant source of the temperature dependence of the correlators. We have found that the low energy contribution is well described by a quasi-particle model and have determined the effective temperature dependent heavy quark mass.

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1. Introduction

Quarkonium correlators in Euclidean time and the corresponding spectral functions at finite temperature have been extensively studied in the past few years in connection with the problem of quarkonium melting in the deconfined phase [1, 2, 3, 4]. Somewhat surprisingly these studies have indicated that quarkonium states can survive up to temperatures as high as $1.6T_c$, contradicting the expectations based on potential models with screening (see e.g. [5, 6, 7]). To get definite answer to the question at which temperature quarkonium states melt and possibly resolve this contradiction a very detailed understanding of the temperature dependence of quarkonium correlators is needed.

Above the deconfinement temperature quarkonium spectral functions contain information not only about quark anti-quark pairs (whether bound or un-bound) but also about scattering states of single heavy quarks in the plasma. The later shows up as a contribution to the spectral function at very low frequency, $\omega \simeq 0$. Therefore this contribution is referred to as the zero mode contribution. It has been realized that the zero mode contribution could be the dominant source of the temperature dependence of the Euclidean correlators in the vector channel [8] as well as in the scalar and axial-vector channels [9] (see also the discussion in Refs. [7, 10]). In this paper we are going to study in detail the zero mode contribution and its relation to in-medium properties of heavy quarks.

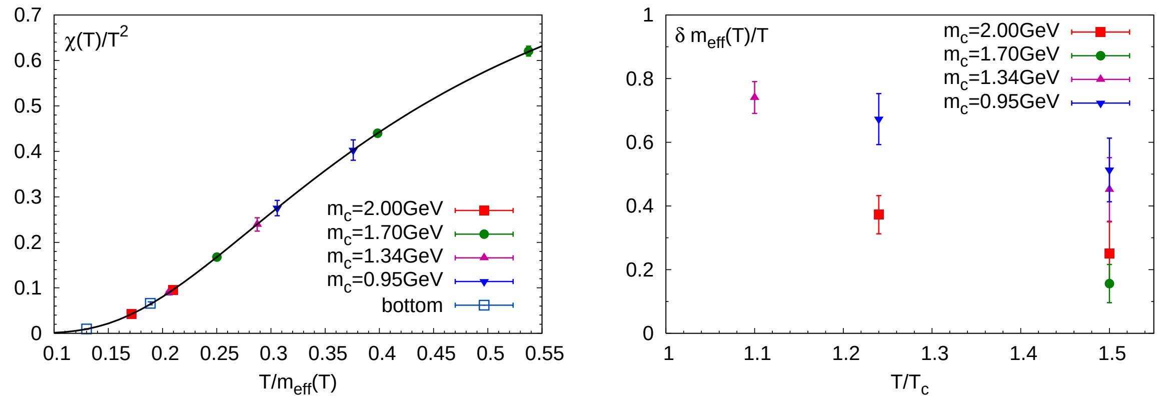


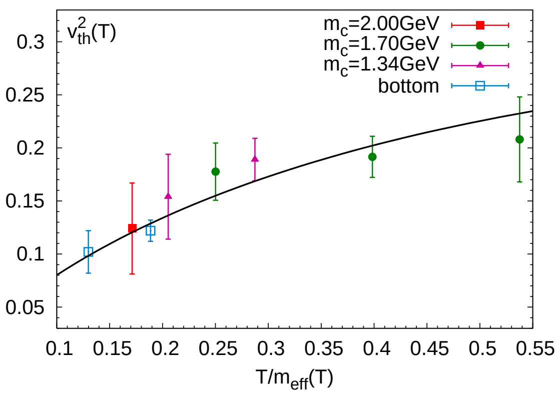
Figure 1. Quark number susceptibility for heavy quarks (left) and the thermal correction to the effective quark mass (right). The line shows the prediction of the quasi-particle model with $m_{\rm eff}(T)$.

2. Numerical Results

We have calculated correlators of local quarkonium operators, $\bar{q}(x)\Gamma q(x)$, in quenched approximation using isotropic lattices with standard Wilson gauge action and non-perturbative clover action for quarks. Calculations have been performed at three different values of the gauge coupling $\beta = 6/g^2 = 6.499,\ 6.640$ and 7.192 corresponding to lattice spacing $a = 0.0451$fm, 0.0377fm and 0.017 fm. The lattice spacing has been set using the Sommer scale $r_0 = 0.5$ fm. We consider several quark masses in the region of the charm quark mass as well as bottom quark. In order to quantify the temperature dependence we have performed calculations both at low temperatures (below the deconfinement temperature) and at high temperatures. At low temperatures the spectral functions can be calculated reliably with the Maximum Entropy Method and the algorithm of Ref. [4]. Using this spectral function we can calculate the reconstructed correlator

$$G^i_{rec}(\tau,T) = \int_0^\infty d\omega \sigma^i(\omega,T^*) \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}. \tag{1}$$

and study the temperature dependence of the ratio $G^i(\tau,T)/G^i_{rec}(\tau,T)$ [1]. Here T^* is some temperature below T_c and i labels the quantum number channel. The trivial temperature dependence due to the integration kernel cancels in this ratio. For sufficiently large quark mass ($m \gg T$) the spectral function can be written as $\sigma^i(\omega,T) = \sigma^i_{\rm low}(\omega,T) + \sigma^i_{\rm high}(\omega,T)$ and similar expression can be written for the correlator (see discussion in Ref. [8]). The high energy part, $\sigma^i_{\rm high}(\omega,T)$ has contribution for $\omega > 2m$ and describes the propagation of quark anti-quark pair. In the free theory $\sigma^i_{\rm low}(\omega,T) = \chi^i(T)\omega\delta(\omega)$, with χ^i being calculated in Ref. [11]. In the presence of interaction the delta function will become a Lorentzian with a width $\sim T^2/m \ll T$ and therefore the low energy contribution $G^i_{\rm low}(\tau,T) \simeq T\chi^i(T)$ is independent of τ to very good approximation. It turns out that this contribution provides the dominant



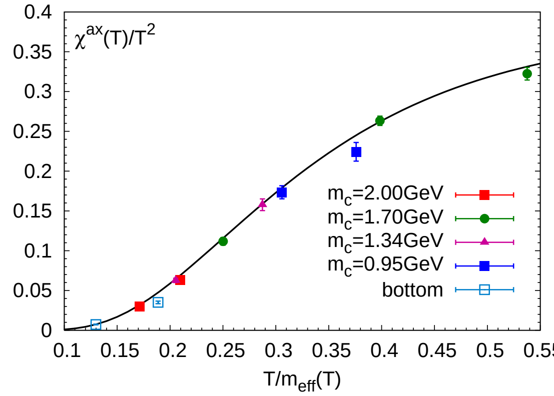


Figure 2. Thermal velocity of heavy quarks (left) and the constant contribution to the axial-vector correlator (right). The lines show the prediction of the quasi-particle model with $m_{\rm eff}(T)$.

source of the temperature dependence of the quarkonium correlators. To see this we have calculated the time derivative of the correlator $G^{i'}(\tau, T)$, where this (almost) constant contribution drops out and studied the temperature dependence of the ratio $G^{i'}(\tau,T)/G^{i'}_{rec}(\tau,T)$. We find that this ratio is close to one, more precisely it deviates from unity by at most 5% and always compatible with one within statistical errors up to temperatures as high as $3T_c$! One may think that this implies the survival of χ_c states up to these temperatures. However, as has been shown in Ref. [7] the melting of the P-states does not lead to significant change in the correlators. and the ratio $G^{i'}(\tau,T)/G^{i'}_{rec}(\tau,T)$ always stays close to one.

Since we expect that almost the entire temperature dependence of the correlator is contained in $G_{\rm low}$ we can assume that $G^i_{\rm high} = G^i_{rec}$ and quantify the low energy part $G^i_{\rm low}$ above T_c using the lattice data. First consider the temporal component of the vector correlator G^{V0}. In this case there is no high energy component and $G^{V0} = T\chi^{V0}(T) \equiv T\chi(T)$, with $\chi(T)$ being the heavy quark number susceptibility. In the free theory χ/T^2 depends only on m/T. Thus matching the free theory expression for $\chi^{V0} = \chi(T)$, calculated in Ref. [11], to the lattice data on G^{V0}, we can define and effective temperature dependent heavy quark mass $m_{\rm eff}(T)$. In Fig. 1 we show the quark number susceptibility as function of $T/m_{\rm eff}$ as well as the effective quark mass. As one can see from the figure the quark number susceptibility is function of $T/m_{\rm eff}$ only. The thermal correction to the heavy quark mass is largest at T_c and monotonically decreases with increasing temperature. For the charm quark mass of $1.7GeV$ it is negligible for $T > 1.5T_c$. Next we consider the the spatial component of the vector correlator. We find that the quasi-particle picture with the effective heavy quark mass determined above describes the data very well. In Fig. 2 we show the ratio $G^{Vs}_{\rm low}(T)/G^{V0}$ which gives an estimate of the thermal velocity squared of the heavy quark $v^2_{\rm th}$. This is because in the free theory $G^{Vs}_{\rm low}(T)/G^{V0} \simeq \int d^3p(p^2/E_p^2)e^{-E_p/T}/\int d^3pe^{-E_p/T} = v^2_{\rm th}$. For $T \gg m$ we have $v^2_{\rm th} \simeq T/m$ but as one can see from the figure it is not a good approximation even for

the b-quark. We also consider the zero mode contribution in the axial-vector channel. The zero mode contribution can be extracted in the same manner as in the vector case. In Fig. 2 we also show $G^{\rm ax}_{\rm low}/T^2 = \chi^{\rm ax}(T)/T^2$ as function of the effective mass m_{eff}/T together with the prediction of the quasi-particle model. The figure clearly shows that the zero mode contribution to the axial-vector channel is the function of the effective quark mass only and is in excellent agreement with the prediction of quasi-particle model.

3. Conclusions

In this paper the temperature dependence of quarkonium correlation function has been discussed. It has been shown that the temperature dependence of the high energy part of the spectral function, for example, the melting of resonances does not lead to a large change in the correlation function. The dominant source of the temperature dependence of quarkonium correlators is the zero mode contribution. This contribution has been studied quantitatively on the lattice for the first time. In general, it is expected that this contribution depends on the temperature and the quark mass. We have found, however, that it is the function of m_{eff}/T only and is well described by a quasi-particle model down to temperatures as low as $1.1T_c$. We have also found that the thermal corrections to the heavy quark mass are small. Since the the quasi-particle mode is so successful in describing the zero mode contribution to the quarkonium correlators it would be of great interest to calculate it systematically in improved perturbation theory [12].

Acknowledgments

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